integral:

$x^2 \rightleftarrows 2x$

$$\int x^a\,dx = \frac{x^{a+1}}{a+1} + C$$

örnek: $\int (x^3+3x+5)\,dx = \frac{x^4}{4} + \frac{3x^2}{2} + 5x + C$

örnek: $\int (6x^2-2x+8x^3-1)\,dx = \frac{6x^3}{3} + \frac{2x^2}{2} + \frac{8x^4}{4} - x + C$

$= 2x^3 - x^2 + 2x^4 - x + C$

örnek: $f''(x) = 12x^2 - 6x - 1$

$f'(1) = 6$
$f(2) = -4$
$f(3) = ?$

$f'(x) = \frac{12x^3}{4} - \frac{6x^2}{2} - x + C$

$f'(x) = 4x^3 - 3x^2 - x + C$

$f'(1) = 4 - 3 - 1 + C = 6$

$\boxed{C=6}$

$\int f'(x) = \int 4x^3 - 3x^2 - x + 6$

$f(x) = \frac{4x^4}{4} - \frac{3x^3}{3} - \frac{x^2}{2} + 6x + C$

$f(x) = x^4 - x^3 - \frac{x^2}{2} + 6x + C$

$f(2) = 16 - 8 - 2 + 12 + C = -4$

$18 + C = -4$

$C = -22$

$f(x) = x^4 - x^3 - \frac{x^2}{2} + 6x - 22$

$f(3) = 81 - 27 - \frac{9}{2} + 18 - 22$

$50 - \frac{9}{2} = \frac{91}{2}$

örnek: $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$

$\frac{x^{-2+1}}{-1} + C = \frac{x^{-1}}{-1} = -\frac{1}{x} + C$

örnek: $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx$

$= \frac{x^{2/3+1}}{\frac{2}{3}+1} + C = \frac{x^{5/3}}{5/3} + C$

$\rightarrow \frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$

$\int d(f(x)) = f(x) + C$

$\int \frac{d}{dx}(f(x))\,dx = f(x) + C$

örnek: $\int (x^2+3x+1)\,dy = x^2y + 3xy + y + C$

önemli durumlar:

$$\int \left(\frac{f(x) - x\cdot f'(x)}{f^2(x)}\right) dx$$

$$\int \left(\frac{x}{f(x)}\right)' dx = \frac{x}{f(x)} + C$$

$$\int (2x\cdot f(x) + f'(x)\cdot x^2)\,dx$$

$$\int (f(x)\cdot x^2)'\,dx = f(x)\cdot x^2 + C$$

örnek: $\left(\int f(x)\,dx\right)' = (5x^2 - 2x - 3)$

$f(x) = 10x - 2$

$x^3 - 2x$ diferansiyelini bulunuz:

$(3x^2 - 2)\,dx$

$u^4 + 3u^3$ diferansiyeli

$(4u^3 + 9u^2)\,du$

⊗ d integrali ($\int$) götürür.

Örnek Soru çözümü:

- $d(3x^2) = 6x$

- $\int (x^2+2x)\,dx$

$\frac{x^3}{3}+\frac{2x^2}{2}+C$

- $x^{-3}\,dx$

$\frac{x^{-2}}{-2}+C = \frac{1}{-2x^2}+C$

- $\int \sqrt{x}\,dx \rightarrow x^{\frac{1}{2}+1}$

$\frac{x^{3/2}}{\frac{3}{2}}+C = \frac{2}{3}x^{3/2}+C$

- $\int\left(\frac{1}{x^2}-\frac{2}{x^3}\right)dx$

$\int (x^{-2}-2x^{-3})\,dx$

$\frac{-x^{-1}}{1}+\frac{2x^{-2}}{-2}+C \rightarrow \frac{-1}{x}+\frac{1}{x^2}+C$

NOT: içerde ise c vardır
dışarda ise aynen yazılır.

- $\frac{d}{dx}\int f(x)\,dx = f(x)$

- $\int f'(x)\,dx = \int \frac{d f(x)}{dx}\,dx = f(x)+C$

- $\frac{d}{dx}\int f(x)\,dx = (x^2+3x+C)\cdot\frac{d}{dx}$

f(x)

$\boxed{f(x) = 2x+3}$

- $\frac{d}{\cancel{dx}}\int x\cdot f(x)\,\cancel{dx} = (x^5+4x^3+C)\frac{d}{dx} \quad f(1)=?$

$\frac{x\cdot f(x)}{x} = \frac{5x^4+12x^2}{x}$

$f(x) = 5x^3+12x$
$f(1) = 5+12 = 17$

Değişken Değiştirme Yöntemi

* $\int (x^2-3x)^5 \cdot (2x-3)\,dx$

$u = x^2-3x$
$du = 2x-3\,dx$

$\int u^5 \cdot du = \frac{u^6}{6} + c$

$\frac{(x^2-3x)^6}{6} + c$

* $\int (f(2x))^3 f'(2x)\,dx = \int u^3 \cdot \frac{du}{2}$

$u = f(2x)$
$du = 2 \cdot f'(2x)\,dx = du$

$\frac{u^4}{8} + c = \frac{(f(2x))^4}{8} + c$

$f'(2x)\,dx = \frac{du}{2}$

* $f(x) = \frac{x^2-2x+4}{\cancel{x-2}} + \frac{x^2-9}{\cancel{x-3}} + x$ $\quad (x-2) \quad \cancel{(x-3)}(x+3)$

$\lim_{x \to 2} f(x) + \lim_{x \to 3} f(x)$

$f(x) = x+2+x+3+x$
$f(x) = 3x+1$

$7+10=17$

BELIRLI INTEGRAL!

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

$f(x)$'in integrali
$F(x)$

* $\int_{-3}^{2} (2x-1)\,dx = x^2+x \Big|_{-3}^{2} = 6-6=0$

* $\int_0^2 (6x^2-1)\,dx = 2x^3-x \Big|_0^2 = 2 \cdot 2^3 - 2$
$14-0=14$

$\frac{6x^3}{3}$
$2x^3$

Özel Durumlar;

* $\int_a^b f'(x)\,dx = f(x)\Big|_a^b = f(b)-f(a)$

* $\frac{d}{dx} \int_a^b f(x)\,dx = 0$

* $\frac{d}{dx} \int_x^{x^2} 2y\,dy = \frac{d}{dx}\left(y^2 \Big|_x^{x^2}\right) = \frac{d}{dx}(x^4-x^2)$
$= 4x^3-2x$

x'in katsayısı olmadığında;

* $\int_2^8 f(x)\,dx = 15$

$\int_{-1}^5 f(x+3)\,dx = ?$

$F(x)\Big|_2^8 = F(8)-F(2)=15$

$F(x+3)\Big|_{-1}^5 = F(8)-F(2)=15$

* $\int_1^5 f(x) = 10$

$\int_0^2 f(2x+1)\,dx = ?$

$F(x)\Big|_1^5 = F(5)-F(1)=10$

$\frac{F(2x+1)}{2}\Big|_0^2 = \frac{F(5)}{2} - \frac{F(1)}{2}$

$= \frac{F(5)-F(1)}{2} = \frac{10}{2} = 5$

* $2x+1=u$

$2 \cdot dx = du$

$dx = \frac{du}{2}$

$\int_1^5 f(u)\frac{du}{2}$

$\frac{F(u)}{2}\Big|_1^5$

Örnekler:

- $\int_0^2 (x^2-5x)^4 \cdot (2x-5)\,dx$

$x^2-5x=u \qquad \int_0^{-6} u^4\,du$

$(2x-5)\,dx=du$

$\frac{u^5}{5}\Big|_0^{-6} = \frac{-6^5}{5} = 0$

- $\int_b^a f(x)\,dx + \int_a^c f(x)\,dx = \int_a^c f(x)\,dx$

$\int_2^5 f(x)\,dx + \int_5^{10} f(x)\,dx = \int_2^{10} f(x)\,dx$

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_2^3 f(x)\,dx - \int_5^3 f(x)\,dx$

$\int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx = \int_2^5 f(x)\,dx$

f tek fonksiyon ise;

$\int_{-a}^{a} f(x)\,dx = 0$

çift $F(x)\Big|_{-a}^{a} = F(a)-F(-a)$

$F(a)-F(a)=0$

- $\int_{-5}^{0} f(x)=4$ f tek

$\int_{-5}^{5} f(x)\,dx=0 \qquad \underbrace{\int_{-5}^{0} f(x)\,dx}_{4} + \underbrace{\int_{0}^{5} f(x)\,dx}_{-4} = 0$

f çift ise (fonksiyon)

- $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

$F(a)-F(-a)$

$F(a)+F(a)=2F(a)$

Parçalı fonksiyon - mutlak değer

- $f(x)\begin{cases} 2x+1, & 1<x \\ 5, & x\geq 1\end{cases} \qquad \int_{-2}^{3} f(x)\,dx=?$

$\int_{-2}^{1}(2x+1)\,dx + \int_1^3 5\,dx$

$x^2+x\Big|_{-2}^{1} + 5x\Big|_1^3$

$(2-2)+15-5$

$=0+10=10$

- $f(x)=\begin{cases} 2x & -2>x \\ 4x-3 & -2\leq x<1 \\ 4 & x\geq 1\end{cases}$

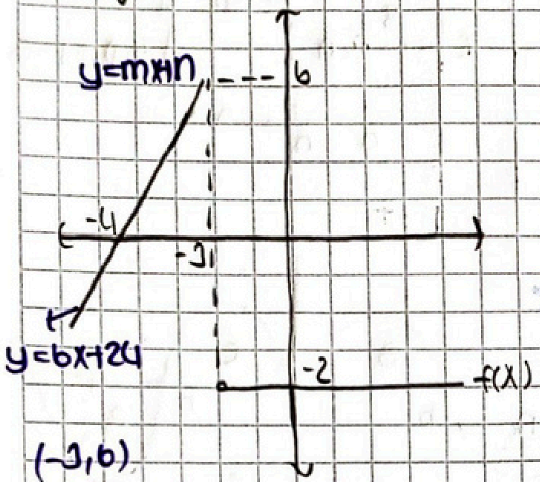
$\int_{-3}^{2} f(x)\,dx=?$

$\int_{-3}^{-2} 2x\,dx + \int_{-2}^{1}(4x-3)\,dx \int_1^2 4\,dx$

$x^2\Big|_{-3}^{-2} + 2x^2-3x\Big|_{-2}^{1} + 4x\Big|_1^2$

$-5-1-14+8-4=-16$

$\int_{-6}^{3} f(x)\,dx = ?$



$(-3,6)$

$(-4,0)$

$m=\frac{-6-0}{-3-(-4)}=6$

$y=6x+1$

$6=-18+n$

$24=n$

$\int_{-6}^{3}(6x+24)\,dx + \int_{-3}^{3}-2\,dx$

$f(x)=\begin{cases} 2x & x\leq 3 \\ 3 & x>3 \end{cases}$

$\int_{-2}^{3} f(x+2)\,dx = ?$

$f(x+2)\begin{cases} 2x+4 & x+2\leq 3 \\ 3 & x+2>3 \end{cases}$

$f(x+2)=\begin{cases} 2x+4 & x\leq 1 \\ 3 & x>1 \end{cases}$

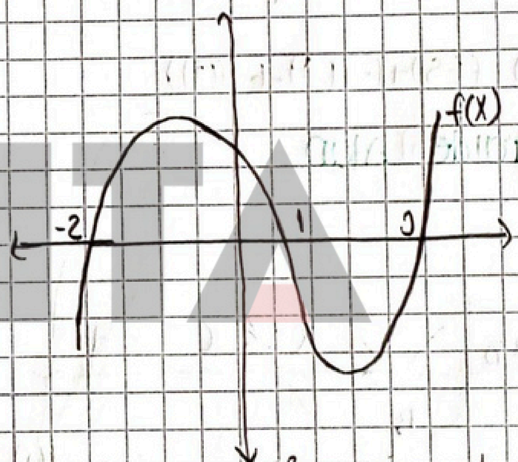
$\int_{-2}^{1}(2x+4)\,dx + \int_{1}^{3} 3\,dx$

önemli! $\int_{-3}^{5}|x-2|\,dx = \int_{-3}^{2}(-x+2)\,dx + \int_{2}^{5}(x-2)\,dx$

$\int_{-2}^{4}|3x-1|\,dx = \int_{-2}^{1/3} -3x+1\,dx + \int_{1/3}^{4} 3x-1\,dx$

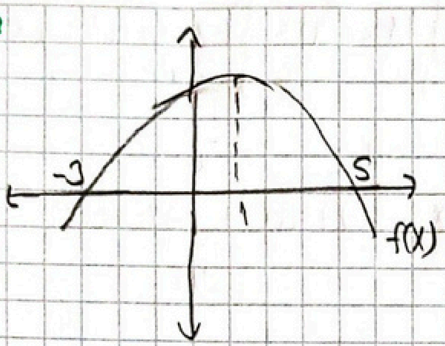
$\int_{-4}^{5}\sqrt{x^2-6x+9}\,dx = \int_{-4}^{5}\sqrt{(x-3)^2} = \int_{4}^{5}|x-3|\,dx$

$\int_{-4}^{3}(-x+3)\,dx + \int_{3}^{5} x-3\,dx$



$\int_{-5}^{6}|f(x)|\,dx = \int_{-5}^{-2} -f(x)\,dx + \int_{-2}^{1} f(x)$

$+ \int_{1}^{3} -f(x)\,dx + \int_{3}^{6} f(x)\,dx$

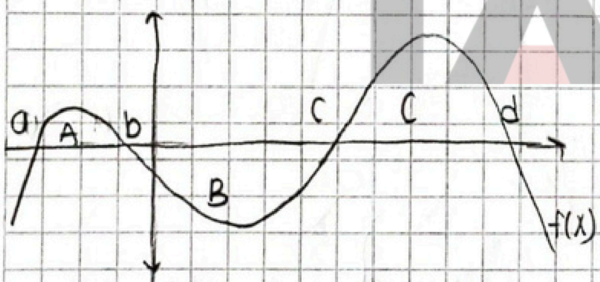


$$\int_{-5}^{7}|f'(x)|\,dx=\int_{-5}^{1}f'(x)\,dx+\int_{1}^{7}-f'(x)\,dx$$

$$f(x)\Big|_{-5}^{1}+-f(x)\Big|_{1}^{7}$$

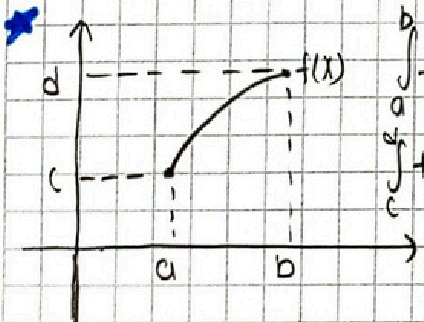
$$f(1)-f(-5)+(-f(7)-(-f(1)))$$

integralde Alan



$$\int_{a}^{b}f(x)\,dx=A \qquad \int_{a}^{d}f(x)\,dx=A-B+C$$

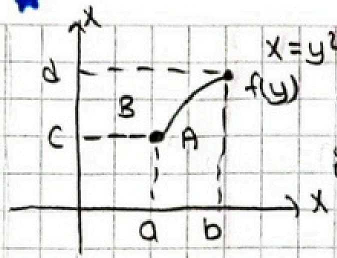
$$\int_{b}^{c}f(x)\,dx=-B \qquad \int_{a}^{d}|f(x)|\,dx=A+B+C$$



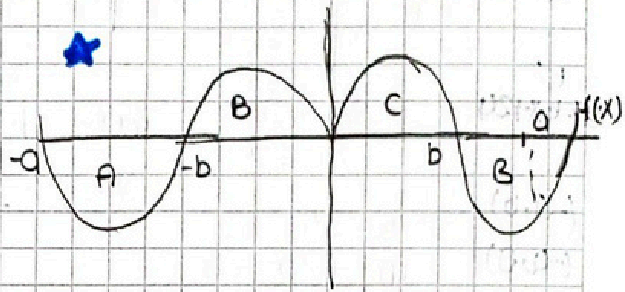
$$\int_{a}^{b}f(x)\,dx=A$$

$$\int_{c}^{d}f^{-1}(x)\,dx=B$$

$x=y^2+1$



$$\int_{c}^{d}f(y)\,dy=B$$



$$\int_{a}^{d}f(y)\,dy=B \qquad \int_{-a}^{a}f(|x|)\,dx=2C-2D$$